Weber's Law and Fechner's Law

Introduction

This handout describes Weber's law and Fechner's law. You are not responsible for the mathematics presented here, but you should know what the terms Weber's law and Fechner's law refer to.

Discrimination Thresholds

Weber's law expresses a general relationship between a quantity or intensity of something and how much more needs to be added for us to be able to tell that something has been added. Experiments designed to find out such things are called *discrimination threshold* experiments, because the observer is asked to tell apart, or discriminate, two things that differ by only a slight increment. The discrimination threshold, then, is the smallest detectable increment above whatever the initial intensity was. You might imagine that this increment is not the same for the entire range of initial intensities that are possible— generally speaking, you're right. The smallest detectable increment typically changes depending on what the starting intensity is.

For instance, if we are trying to discriminate which of two flashes of light lasts longer, we may be able to tell the difference between a short flash and a slightly longer one. But if the same slight difference in duration is added to a very long flash, we are less able to tell the difference between the slightly-longer long flash from the long flash by itself. Similarly, for a particular combination of target and masker sounds, our ability to detect a tone in the presence of a masker depends on how intense the masker is. If the masker is not very intense, we don't have to increase the intensity of the tone very much to be able to detect it. If the masker is more intense, the same small increase in intensity will not make the tone audible anymore. We won't be able to detect it unless we make it considerably more intense.
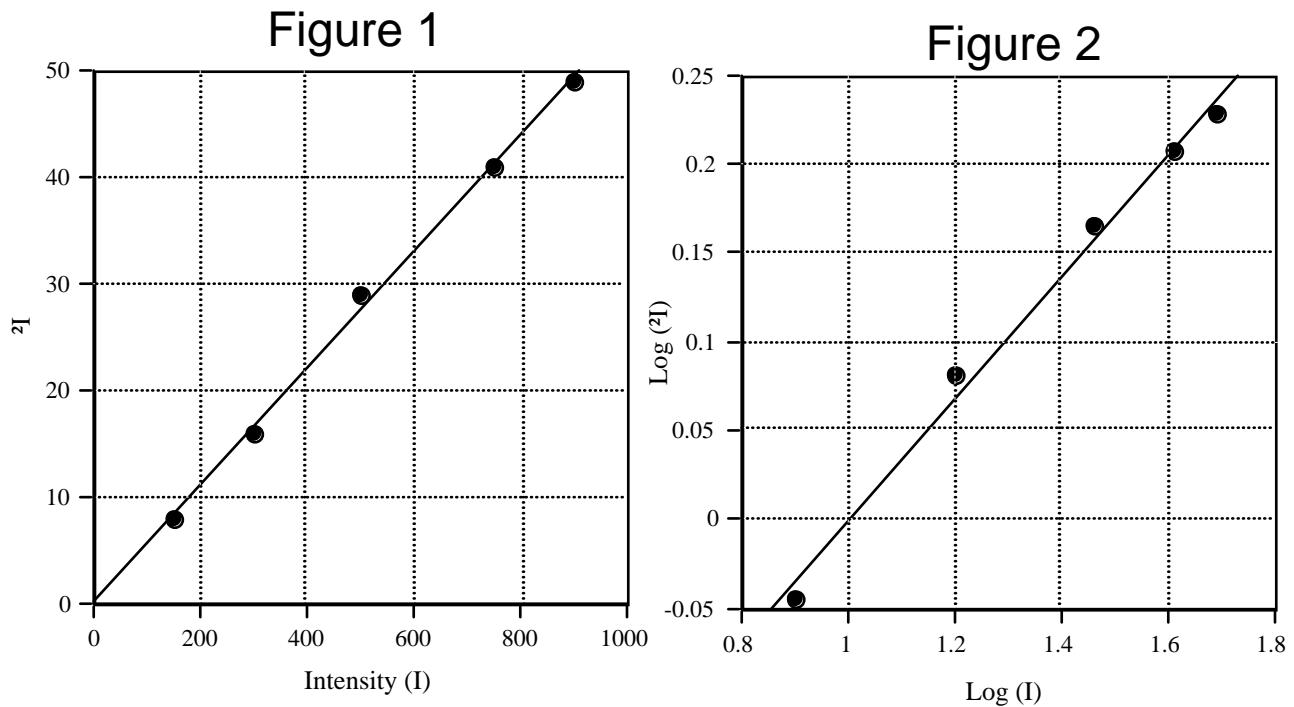
Weber's law

This general relationship between the initial intensity of something and the smallest detectable increment is exactly what Weber noticed and formalized into "Weber's Law". The discrimination threshold, or the threshold for detecting an increment in the quantity or intensity of something, changes depending on how much there is before we add the increment. Weber's law is a hypothesis about how this threshold change happens. Let's call the initial intensity (the intensity before adding the increment) I, and let's call the amount needed to detect a difference ΔI. (Often, the specific letter that researchers choose bears some relation to the stimulus dimension being discriminated—if we are interested in duration discrimination, we might have D and ΔD.)

In a discrimination experiment, then, we are interested in measuring ΔI as a function of I. That is, we want to find the discrimination threshold ΔI such that a stimulus with an intensity $I+\Delta I$ is just discriminable from a stimulus of intensity I. Weber's law characterizes how the ΔI we measure depends on I. It states that

$$\Delta I = K_w I$$

for some constant K_W. The constant K_W is called the Weber Fraction. You may notice the similarity between this equation and the equation for a straight line, $y = m * x + b$. Weber's law expresses the equation for a straight line, with the slope m being the Weber Fraction, K_W and the y-intercept b being zero. Likewise, I plays the part of x and ΔI plays the part of y. Thus, if Weber's law holds, we expect the data to graph as a straight line through the origin, as shown in Figure 1.

Figure 1

Figure 2

Other ways of expressing Weber's law

There are other ways of expressing Weber's law which might allow a more convenient visual analysis of whether or not Weber's law holds for a set of data. The first one described below requires you to divide the measured difference thresholds ΔI by the intensity I. The second one requires you to convert the discrimination thresholds and the base intensity into logarithms:

$$\frac{\Delta I}{I} = K_w .$$

In this formulation of Weber's law, if $\Delta I/I$ is plotted on the *y*-axis and *I* is plotted on the *x*-axis, you should see a horizontal straight line with intercept K_W if Weber's law holds. Another way of saying this is that no matter what intensity *I* we present to the observer, if Weber's law holds, we should measure a discrimination threshold ΔI that always results in the same number when we divide it by *I*. Thus, the fraction $\Delta I/I$ is a *constant* (K_W, the Weber Fraction) which does not change for different values of *I*. Thus,

$$\log(\Delta I) = \log(I) + \log(K_w) .$$

This formula is useful when *I* can take on a large range of values. If Weber's law holds, $\log(\Delta I)$ should plot as a straight-line function of $\log(I)$, with slope 1 and intercept K_W. In Figure 2 above, the data plotted in Figure 1 were transformed to logs and replotted. You can see that the data still fall along a straight line.

Why Weber's law is useful

As we have already seen, Weber's law is a useful way to summarize the relation between the discrimination threshold ΔI and the base intensity *I*. In general, this relation holds true for many

different physical dimensions (but not always—see below). If we know that Weber's law holds for two dimensions, we can compare our sensitivity to changes along those dimensions by comparing the Weber fractions. The Weber fraction is often expressed as a percentage. A Weber fraction of 1% indicates a fairly high sensitivity to increments, while a Weber fraction of 15% is rather poor and indicates lower sensitivity to increments.

When performing discrimination threshold experiments, the first analysis is often aimed at finding out whether or not Weber's law holds for that set of discriminations. If it does, we then want to find out what the Weber fraction is. By now, you can probably see that if we know Weber's law holds and we know what the Weber fraction is, we can *predict* what the discrimination threshold should be—it's just the Weber fraction times the base intensity.

How general is Weber's law?

You may have already suspected that Weber's law might not hold for absolutely all values of *I*. In fact, it's often the case that for very small values of *I*, Weber's law starts to break down. Here is a generalized formulation of Weber's law that can accommodate the most common deviation from the simple form of the law:

$$\Delta I = K_w (I + I_0).$$

$K_w I_O$ is the absolute threshold—the smallest intensity of a given physical dimension we can reliably detect. You can see that when *I* is large, the difference threshold ΔI approaches $K_w * I$, which is the usual formulation for Weber's law. When *I* is near threshold, however, ΔI approaches $K_w * I_O$. This means that when *I* takes on values near threshold, the difference threshold ΔI is more influenced by the absolute threshold (which is approximately constant, ignoring a bit of noise) than it is by the changing value of *I*.

Fechner's law

Fechner's law provides an explanation for Weber's law. Fechner's explanation has two parts. The first part is that two stimuli will be discriminable if they generate a visual response that exceeds some threshold. The second part is that the visual response *R* to an intensity *I* is given by the equation

$$R = \log(I).$$

This relation is plotted in the next graph. Suppose we call the change in response necessary for discrimination "one". Then by Fechner's law the two intensities indicated by arrows will be just discriminable. The discrimination threshold ΔI is given by the distance on the x-axis between *I* and *I*+ΔI.

$$I \quad I+\Delta I$$
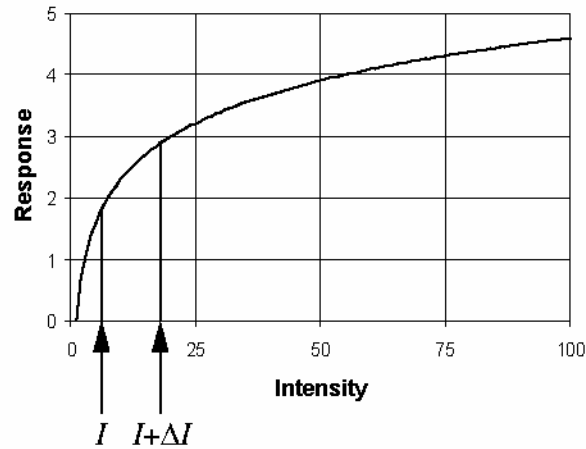
If you use Fechner's law and the graph to find the discrimination threshold for a stimulus more intense than the *I* shown, you will you will discover that the discrimination threshold is larger. This is consistent with Weber's law, which predicts that the discrimination threshold grows as we increase the base intensity *I*.

Mathematical relation between Fechner's law and Weber's law

(You are not responsible for this math. I provide it for the curious.)

Fechner's law predicts that two lights will be just discriminable if the responses they generate differ by a constant amount. Let us call this amount ΔR. So Fechner's law says that when ΔI represents discrimination threshold,

$$\Delta R = \log(I + \Delta I) - \log(I).$$

Using the rules for logarithms, we can re-write this as

$$\Delta R = \log\big((I + \Delta I)/I\big).$$

Now raise the constant *e* to both sides of this equation:

$$e^{\Delta R} = e^{\log((I+\Delta I)/I)}.$$

Since ΔR is a constant, we can simply define a new constant $K = e^{\Delta R}$. Since $e^{\log(x)} = x$ for any *x* (by the definition of a logarithm), we can re-write our equation as

$$K = (I + \Delta I)/I.$$

Now multiply both sides by *I*:

$$KI = I + \Delta I.$$

Now subtract *I* from both sides:

$$KI - I = \Delta I.$$

Now regroup the terms on the left:

$$(K - 1)I = \Delta I.$$

Now define

$$K_w = K - 1 = e^{\Delta R} - 1.$$

Then we have Weber's law:

$$\Delta I = K_w I,$$

for some constant K_w. (And we have even shown how K_w depends on ΔR.) The result of this math is that if Fechner's law holds, then we can predict Weber's law.

Final notes for the brave of heart

(You are not responsible for this next paragraph.)

The visual response that plays a role in Fechner's law is closely related to the perceptual response that played a role in our discussion of the theory of signal detection. If you are curious, stop by during office hours and we can chat about the relation in more detail. I also note that there are conditions under which Weber's law does not hold. Again, if you are curious please come ask me to tell you more.